

A Replenishment Inventory Model for Items Under Time-Varying Demand Rates Considering Trade Credit Period and Cash Discount for a Finite Time Horizon

(Model Penambahan Inventori Bila Kadar Permintaan Berubah Terhadap Masa Dengan Bayaran Tertangguh dan Diskaun Tunai Dibenarkan untuk Masa Terhingga)

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ABSTRACT

Many researchers have developed various economic ordering quantity models by assuming an infinite time horizon and constant demand rate. However due to rapid technological advancement, shorter product life cycle and severe competition, those assumptions are no longer realistic. In this paper, we complement that shortcoming by considering an inventory model that satisfies a continuous time-varying demand rate for a finite time horizon when trade credit period and unit cash discount are allowed. The time horizon consists of n different cycles with equal or different cycles length. The trade credit period was assumed to be proportional to the cycle length. We developed mathematical models and presented a numerical example to support the effectiveness of these models.

Keywords: Cash discount; delay payment; finite horizon; inventory; time-varying demand

ABSTRAK

Ramai penyelidik telah membangunkan pelbagai model Kuantiti Pesanan Ekonomi dengan andaian masa perancangannya tak terhingga dan kadar permintaannya adalah tetap. Bagaimanapun perkembangan pesat teknologi, masa produk yang singkat dan persaingan yang hebat, andaian itu tidak lagi realistik. Kertas ini akan menghilangkan kekurangan itu dengan membina model inventori bila kadar permintaan berubah terhadap masa untuk masa terhingga bila bayaran tertangguh dan diskaun tunai dibenarkan. Masa perancangannya mengandungi n kitaran dengan tempoh yang sama atau berbeza. Tempoh bayaran tertangguh diandaikan berkadaran dengan tempoh bagi setiap kitaran. Kami telah membina model matematikanya dan memberikan contoh berangka untuk menyokong keberkesanan model tersebut.

Kata kunci: Bayaran tertangguh; diskaun tunai; inventori; masa terhingga; permintaan berubah terhadap masa

INTRODUCTION

In the traditional economic order quantity (EOQ) model, it is implicitly assumed that the retailer must pay for the ordering items when they are received. In reality however, we find that the supplier offer the retailer a delay in payment for fixed time period (or a trade credit period) and do not charge any interest during this period. This incentive will reduce the retailer's cost of holding stock because it reduces the amount of capital invested in stock for that period. Consequently it will motivate the retailer to enlarge his current order size and which in turn leads to a reduction in purchasing cost and ordering cost. In order to encourage the retailer to pay for his purchase quickly, sometimes the supplier may offer a cash discount. For example, the supplier offer a 5% discount off the unit product price if the payment is made within 15 days; otherwise the full price of the product is due within 30 days. During the trade credit period, the retailer can sell the goods, accumulate revenue and earn interest.

Goyal (1985) were amongst the first to develop an economic ordering quantity (EOQ) model under the

condition of permissible delay but ignored the difference between the selling price and purchase cost. Dave (1985) extended the Goyal's model by assuming the selling price is higher than its purchase price. Aggarwal and Jaggi (1995) extended Goyal's model to allow for deteriorating items and Jamal et al. (1997) further extended the model to allow for shortages. Ouyang et al. (2002) and Chang (2002) developed inventory models by considering a cash discount and delay payment.

The common assumption in all the above model is constant demand rate over infinite horizon. However in many real-life situations, those assumptions might not be realistic due to shorter product life cycle, seasonal demand and severe competition. The demand rate normally increases especially in the early stage of product life or decreases at the final stage for example before the product is taken out from the production line. To overcome this problem, in this paper we extended the previous model to the case of continuous time-varying demand rates over finite time horizon planning when credit period or unit cash discount are allowed. In this model, the time horizon

was divided into n different cycles and the credit period was proportional to the cycle length. The cycle length could be equal or different. We developed mathematical models and numerical examples were given to illustrate the effectiveness of the models.

ASSUMPTIONS AND NOTATION

The following common assumptions in the literature were used.

1. The demand rate is continuous and time dependent.
2. Shortages are not allowed.
3. Replenishment is instantaneously.
4. The entire time horizon is finite.

In this paper, we adopted the following notation.

1. $D(t)$ is the demand rate per unit time with $D(t) > 0$.
2. K is the ordering cost per order.
3. h is the holding cost per unit per unit time excluding interest charges.
4. M_{i+1} is the credit period for the cycle (t_i, t_{i+1}) where $i = 0, 1, \dots, n - 1$.
5. I_e is the interest earned per dollar per unit time.
6. I_c is the interest charges per dollar in stocks per unit time.
7. c is the purchasing cost per unit.
8. p is the selling price per unit with $p > c$.
9. r is the unit cash discount.
10. T_H is the time horizon.

MODEL FORMULATION

We develop the mathematical formulation for the models. The graphical representation of the inventory systems for unequal lot size (or unequal period) with $n = 3$ is depicted in Figure 1. In Figure 1, $I_i(t)$ represents the remaining stock on-hand in the first cycle where $I_{i+1} = \int_t^{t_{i+1}} D(u) du$ while, $S_{i+1}(t)$ is the selling items also in the first cycle where $S_{i+1}(t) = \int_t^{t_i} D(u) du$ for $i = 0, 1, \dots, n - 1$.

THE TOTAL RELEVANT COST WITH CONSIDERING DELAY PAYMENT

First we assumed that $t_i + M_{i+1} < t_{i+1}$, where M_{i+1} is the length of the credit period in the cycle i . During the credit period, the retailer can start to accumulate revenues on the sales or use of the product, and earn interest on that revenue. The cost of holding stock will be reduced because it reduces the amount of capital invested in stock for the duration of credit period. At the end of this period, the retailer pays off all ordering units, keep profits and start paying for the interest charges or opportunity cost on items in stock. The cost for i th cycle are:

1. Ordering cost, K (1)

2. Holding cost is equal to,

$$h \int_t^{t_{i+1}} \left[\int_t^{t_{i+1}} D(u) du \right] ds.$$

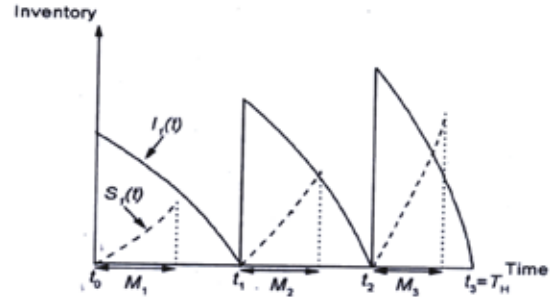


FIGURE 1. Graphical representation of the inventory systems

By changing the order of integration, we have

$$h \int_{t_i}^{t_{i+1}} \left[\int_{t_i}^t D(u) ds \right] du = h \int_{t_i}^{t_{i+1}} (t - t_i) D(u) du. \quad (2)$$

3. Interest earned,

$$pI_e \int_{t_i}^{t_i + M_{i+1}} \int_{t_i}^t D(u) du ds$$

By changing the order of integration, we have

$$pI_e \int_{t_i}^{t_i + M_{i+1}} (t_i + M_{i+1} - t) D(u) du \quad (3)$$

4. Interest charges (opportunity cost)

$$cI_c \int_{t_i + M_{i+1}}^{t_{i+1}} \int_{t_i}^{t_{i+1}} D(u) du ds.$$

Again, by changing the order of integration we have

$$cI_c \int_{t_i + M_{i+1}}^{t_{i+1}} (t - t_i - M_{i+1}) D(u) du. \quad (4)$$

If $t_i + M_{i+1} \geq t_{i+1}$, then we do not incur any interest charges but we have extra interest earned which is given by

$$pI_e (t_i + M_{i+1} - t_{i+1}) \int_{t_i}^{t_{i+1}} D(u) du. \quad (5)$$

So we have the total interest earned as

$$(6)$$

It follows that the total relevant cost for n batches if $t_i + M_{i+1} \leq t_{i+1}$ is:

$$TRC_1(n, t_0, t_1, \dots, t_n) = nK + \sum_{i=0}^{n-1} \left[h \int_{t_i}^{t_{i+1}} (t - t_i) D(u) du + cI_c \int_{t_i+M_{i+1}}^{t_{i+1}} (t - t - M_{i+1}) D(u) du - pI_e \int_{t_i}^{t_i+M_{i+1}} (t_i + M_{i+1} - t) D(u) du \right]. \tag{7}$$

Similarly, for $t_i + M_{i+1} \geq t_{i+1}$, then we have

$$TRC_2(n, t_0, t_1, \dots, t_n) = nK + \left[\sum_{i=0}^{n-1} h \int_{t_i}^{t_{i+1}} (t - t_i) D(u) du - pI_e \left(\int_{t_i}^{t_{i+1}} (t_{i+1} - t) D(u) du + (t_i + M_{i+1} - t_{i+1}) \int_{t_i}^{t_{i+1}} D(u) du \right) \right]. \tag{8}$$

From our models, in each cycle we have two possibilities either M_{i+1} fall inside or outside the interval (t_i, t_{i+1}) . In this numerical analysis, we would like to consider a case where M_{i+1} always inside the cycle interval (t_i, t_{i+1}) by letting $M_{i+1} = (\alpha - 1) t_i + (1 - \alpha) t_{i+1}$ where $0 < \alpha < 1 \dots$

For all values of M_{i+1} , if $D(t) = a + bt$ then we have

$$TRC_1(n, t_0, t_1, \dots, t_n) = nK + \frac{1}{6} \sum_{i=0}^{n-1} \left[h(t_i - t_{i+1})^2 (3a + bt_i + 2bt_{i+1}) - pI_e \left((\alpha - 1)t_i + (1 - \alpha)t_{i+1} \right)^2 \left\{ 3a + 3bt_i + b \left((\alpha - 1)t_i + (1 - \alpha)t_{i+1} \right) \right\} + cI_c \left(t_i - t_{i+1} + (\alpha - 1)t_i + (1 - \alpha)t_{i+1} \right)^2 \left\{ 3a + bt_i + 2bt_{i+1} + b \left((\alpha - 1)t_i + (1 - \alpha)t_{i+1} \right) \right\} \right]. \tag{9}$$

and

$$TRC_2(n, t_0, t_1, \dots, t_n) = nK + \frac{1}{6} \sum_{i=0}^{n-1} (t_i - t_{i+1}) \left[h(t_i - t_{i+1}) (3a + bt_i + 2bt_{i+1}) - 3apI_e \left\{ 2 \left((\alpha - 1)t_i + (1 - \alpha)t_{i+1} \right) + t_i - t_{i+1} \right\} - bpI_e \left\{ t_i^2 + t_i t_{i+1} - 2t_{i+1}^2 + 3(t_i + t_{i+1}) \left((\alpha - 1)t_i + (1 - \alpha)t_{i+1} \right) \right\} \right], \tag{10}$$

respectively.

THE TOTAL RELEVANT COST WITH CONSIDERING DELAY PAYMENT AND UNIT CASH DISCOUNT

We have the similar total cost structure for this problem. However in this paper we only considered the total unit cost discount from unsold items and not from all ordered

items as in Chang (2002). Consequently, we have to subtract the total unit cost discount for the unsold items from time $t_i + M_{i+1}$ to t_{i+1} . For each cycle, the unit cost discount is given by

$$rc \int_{t_i+M_{i+1}}^{t_{i+1}} D(u) du.$$

It follows that the total unit cost discount for n -batch is

$$rc \sum_{i=0}^{n-1} \int_{t_i+M_{i+1}}^{t_{i+1}} D(u) du.$$

If $D(t) = a + bt$ then our new total inventory cost become

$$TRC_{dis}(n, t_0, t_1, \dots, t_n) = TRC_1(n, t_0, t_1, \dots, t_n) - \frac{cr\alpha}{2} \sum_{i=0}^{n-1} \left[(t_{i+1} - t_i) (2a + b\alpha t_i - b(\alpha - 2)t_{i+1}) \right]. \tag{11}$$

SOLUTION PROCEDURE AND NUMERICAL EXAMPLE

The above total relevant cost, $TRC_j, j=1,2$ is a function in n, t_0, t_1, \dots, t_n . For every n , the necessary condition for TRC_j to be minimum is $\frac{d(TRC_j)}{dt_i} = 0$. By using the iterative equation from the necessary condition, and following Balkhi & Bukherouf (2004) and Omar (2006) procedures we find the minimum of the total relevant cost. For example if $D(t) = a+bt$, then from (9) we have:

$$\left[-\frac{1}{2}bh - bca^2 I_c \left(1 - \frac{1}{2}\alpha \right) - bp\alpha I_e \left(\frac{1}{2} - \alpha + \frac{1}{2}\alpha^2 \right) \right] t_{i+1}^2 + \left[-ah - bcra(\alpha - 1) - cI_c \alpha^2 (a - bt_i + b\alpha t_i) + pI_e (\alpha - 1)^2 (a + bt_i + b\alpha t_i) \right] t_{i+1} + \frac{1}{2} \left[ah(4t_i - 2t_{i-1}) + bht_i(3t_i - 2t_{i-1}) + 2bcra(2t_i - t_{i-1})(\alpha - 1) \right] + \frac{1}{2} cI_c \alpha^2 \left[a(4t_i - 2t_{i-1}) + b(3t_i^2 + 2t_i t_{i-1}(\alpha - 2) - t_{i-1}^2(\alpha - 1)) \right] - \frac{1}{2} pI_e (\alpha - 1)^2 \left[a(4t_i - 2t_{i-1}) + b(3t_i^2 + 2\alpha t_i t_{i-1} - t_{i-1}^2(\alpha + 1)) \right] = 0. \tag{12}$$

From the above quadratic equation, we can rewrite the iterative equation of t_{i+1} in terms of t_{i+1} and t_i . Now with $t_0 = 0$, for a known and feasible value of t_i where $0 < t_i < T_H$, the value of t_2 is easily deductible from the above iterative equation. We only accept the positive value of t_2 . From t_2 will leads to t_2 until t_n . All t_{iS} are optimal if $t_n = T_H$. The computer algorithm for this search is as follows:

1. Start with any value of n ($n > 1$).
2. For a finely graded spectrum of values of x in $(0, T_H)$,
- 2a. Let $t_0 = 0$.
- 2b. Let $t_1 = x$.
- 2c. Generate t_2, t_3, \dots until t_n .
- 2d. If $|t_n - T_H| \approx 0$, go to step 3. Else repeat step 2.
3. Set $t_1 = x$ and $t_n = T_H$. Compute $TRC_i(n)$ using the newly found optimal solution of $(t_1, t_2, \dots, t_{n-1})$.

We assumed that the total relevant cost, $TRC_i(n)$ is convex in n . The optimal value of n^* is the first value of n that satisfies $TRC_i(n^*) \leq TRC_i(n^* - 1)$ and $TRC_i(n^*) \leq TRC_i(n^* + 1)$. A similar procedure was applied to $TRC_{dis}(n)$.

NUMERICAL EXAMPLE

Several numerical examples were carried out in order to demonstrate the effectiveness of the models. As mentioned earlier, we only consider the cases where all M_{i+1} fall inside the cycle interval (t_i, t_{i+1}) .

Example 1. Let us consider an inventory system with the following data: $D(t) = a + b_t$ with $a = 0, T_H = 1, K = 200, h = 4, c = 20, p = 80, I_e = 0.04, I_c = 0.04, I_r = 0.12$ and $\alpha = 0.7$. Table 1(a) gives the total relevant costs for several values of n when the values of $b = 1000, 2000, 3000$. It is clearly shown that $TRC_i(n)$ is convex in n . Table 1(b) gives the detail of the optimal total relevant cost for each case. For example, when $b = 2000$ the minimum batches is 4 with the minimum total relevant cost is 1442.562. However, the minimum batches and total relevant cost for all cases when the replenishment periods are the same is (3; 1068.193), (4; 1479.467) and (5; 1798.928), respectively.

Example 2. Let us reconsider Example 1 but with a shorter credit period and unit cash discount where $\alpha = 0.8$ and unit cash discount $r = 0.01$. Table 2(a) gives the the total relevant costs with considering unit cost discount for several values of n when the values of $b = 1000, 2000, 3000$. Similarly, it is clearly shown that $TRC_{dis}(n)$ is convex in n . Table 2(b) gives the detail of the optimal total relevant cost for each case. For example, when $b = 2000$ the minimum batches is 4 with the minimum total relevant cost is 1333.045. The minimum batches and total relevant cost for all cases when the replenishment periods are the same is (3; 1025.363), (4; 1375.533) and (5; 1626.432) respectively. As expected, by comparing with the optimal total relevant cost in the Table 2(b), this policy will encourage the retailer to settle the payment quickly. For example when $b = 2000$, the values of M_{i+1} from Table 2(b) are (0.068058, 0.050521, 0.042960, 0.038461) compare to the values from Table 1(b) as (0.100904, 0.076024, 0.064889, 0.058184).

TABLE 1. (a). Total relevant cost, for different values of and

b	n					
	1	2	3	4	5	6
1000	1969.733	1126.028	1047.324	1121.281	1250.015	1404.359
2000	3739.467	1852.057	1494.648	1442.562	1500.029	1608.717
3000	5509.200	2578.085	1941.971	1763.843	1750.044	1813.076

TABLE 1. (b). Minimum total relevant cost with respect to different values of b

b	n	t_0	t_1	t_2	t_3	t_4	t_5	$TRC_1(n)$
1000	3	0	0.417274	0.731660	1	-	-	1047.324
2000	4	0	0.336346	0.589758	0.806055	1	-	1442.562
3000	5	0	0.285399	0.500427	0.683962	0.848530	1	1750.044

TABLE 2. (a) Total relevant cost, $TRC_{dis}(n)$ for different values of b and n with considering unit cost discount

b	n					
	1	2	3	4	5	6
1000	1996.267	1097.645	1001.260	1066.522	1190.093	1341.019
2000	3792.533	1795.290	1402.521	1333.045	1380.185	1482.039
3000	5588.800	2492.935	1803.781	1599.567	1570.278	1623.058

TABLE 2. (b) Minimum total relevant cost with respect to different values of b

b	n	t_0	t_1	t_2	t_3	t_4	t_5	$TRC_1(n)$
1000	3	0	0.421133	0.733949	1	-	-	1001.260
2000	4	0	0.340291	0.592896	0.807695	1	-	1333.045
3000	5	0	0.289311	0.503943	0.686420	0.849774	1	1570.278

CONCLUSION

In this paper, we consider an inventory model under trade credit through the development of mathematical model. This paper relaxes the common assumptions of fixed demand rate and infinite time horizon for example for a seasonal product. This is a general model for a time-varying demand rate. The similar approach can be extended to another problem for example by considering deteriorating item while stock on-hand, stock-dependent demand or vendor-buyer coordination.

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